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Two-Body Relaxation in Cosmological Simulations

James Binney[1] and Alexander Knebe[1,2],
[1] *Theoretical Physics, 1 Keble Road, Oxford OX1 3NP*
[2] *Centre for Astrophysics & Supercomputing, Swinburne University, Mail # 31, PO Box 218, Hawthorn, VIC 3122, Australia*

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ABSTRACT
It is logically possible that early two-body relaxation in simulations of cosmological clustering influences the final structure of massive clusters. Convergence studies in which mass and spatial resolution are simultaneously increased, cannot eliminate this possibility. We test the importance of two-body relaxation in cosmological simulations with simulations in which there are two species of particles. The cases of two mass ratios, $\sqrt{2}:1$ and $4:1$, are investigated. Simulations are run with both a spatially fixed softening length and adaptive softening using the publicly available codes GADGET and MLAPM, respectively.

The effects of two-body relaxation are detected in both the density profiles of halos and the mass function of halos. The effects are more pronounced with a fixed softening length, but even in this case they are not so large as to suggest that results obtained with one mass species are significantly affected by two-body relaxation.

The simulations that use adaptive softening are less affected by two-body relaxation and produce slightly higher central densities in the largest halos. They run about three times faster than the simulations that use a fixed softening length.

Key words: methods: numerical – galaxies: formation – cosmology: theory

1 INTRODUCTION

The Cold Dark Matter (CDM) model for the formation of cosmological structure has enjoyed considerable success over the last decade and a half. Recent results from the PSCz survey (Hamilton & Tegmark 2002) underline the ability of this model to account successfully for observations of the distribution of galaxies and the structure of rich galaxy clusters within constraints set by measurements of the cosmic background radiation, the theory of primordial nucleosynthesis and observations of distant type Ia supernovae.

By contrast with these successes where large-scale phenomena are concerned, there are serious doubts as to whether the CDM model is compatible with the internal structures of galaxies (e.g., Spergel & Steinhardt 2000). All these difficulties can be traced to the cuspy central density profiles of dark matter halos. This phenomenon was first identified by Navarro, Frenk & White (1996), who concluded that the central density diverged with radius as r^{-1}. Subsequent work by Moore et al. (1998, 1999), Ghigna et al. (2000) and Klypin et al. (2001) suggests that the divergence is even steeper: $\rho \sim r^{-1.5}$.

On sufficiently small scales, discreteness effects must cause deviations between density profiles in simulations and those of real dark-matter halos, in which the particle mass is believed to be tiny. Determining the smallest scale on which numerical simulations are trustworthy has proved difficult and has given rise to some controversy. Kravtsov et al. (1998), Ghigna et al. (2000) and Klypin et al. (2001) have approached this problem by progressively increasing the resolution of simulations started from initial conditions that sampled the same underlying density field. Kravtsov et al. (1998) argued that their simulations were converging on a central density profile of halos that was less cuspy than the NFW profile. Subsequently, Ghigna et al. (2000) and Klypin et al. (2001) showed that simulations converge on profiles more cuspy than the NFW profile if one simultaneously increases both the mass and the spatial resolution; Kravtsov et al. (1998) had increased only the spatial resolution.

While the results of Ghigna et al. (2000) and Klypin et al. (2001) suggest that real dark matter halos should have very cuspy centres as Moore et al. (1998) originally concluded, they do not establish this claim beyond all reasonable doubt. The reason is that when structure forms bottom-up, as in the CDM paradigm, and mass and spatial resolution are increased together, the first virialized systems are few-body systems, regardless of the resolution employed. In such systems the two-body time is comparable to the dynamical time, and two-body interactions tend to make the core denser, and less vulnerable to subsequent tidal destruction. As clustering progresses, one can imagine that the dynamics is dominated by the interaction of quasi-particles formed by first-generation systems. The early clustering of these quasi-particles is again heavily influenced

by two-particle interactions, and leads to the formation of a new generation of quasi-particles, each of which is made up of quasi-particles of the previous generation. Thus one can imagine the dynamics right up to the macroscopic scale being influenced by the two-body interactions that took place between the real simulation particles as the density field first went non-linear. Convergence tests of the type performed by Ghigna et al. (2000) and Klypin et al. (2001) would be powerless to expose discreteness effects if this picture were valid, because a low-resolution simulation would simply enter the evolutionary sequence of a high-resolution simulation at the stage proper to the mass of its simulation particles.

An indication that we should worry about the possibility just described comes from the observation that cuspy profiles are only obtained when the smoothing kernel used to calculate gravitational forces from particle positions is hard. Now, Fig. 10 of Knebe, Green & Binney (2001) shows that for the values of the softening parameter that yield cuspy profiles, the net gravitational force on a particle may be dominated by the force from its nearest neighbour prior to the virialization of the smallest modelled scales. This circumstance is worrying, because the essence of collisionless dynamics is that the forces acting on each particle reflect the mean density field rather than the chance location of a neighbour. Moreover, large forces from neighbours can affect the dynamics of particles more strongly at early times, when random velocities are small and thus force contributions are relatively slowly changing.

The standard way of determining the significance for a simulation of two-body relaxation is to include particles of more than one mass: if the simulation is collisionless, the final distributions of the particles will be independent of mass, whereas the more massive particles will tend to sink to the bottoms of potential wells if two-body relaxation is significant. Since no results from multi-mass cosmological simulations have appeared, we present here the results of such experiments. The experiments were conducted with two radically different simulation codes, the tree-code GADGET (Springel, Yoshida & White 2001) and the adaptive multigrid code, MLAPM (Knebe et al. 2001). We detect the effects of two-body relaxation in results produced with both codes. However, the magnitude of the effect is not so large as to suggest that the cuspiness of dark-matter halos is generated by two-body relaxation. We argue instead that cuspiness arises from violent relaxation, and will invariably arise when gravity causes particles to cluster collisionlessly in a cosmological context.

2 INITIAL CONDITIONS

Cosmological simulations are invariably initialized by using the Zel'dovich approximation to displace particles from an initial approximately homogeneous distribution (e.g., Efstathiou et al. 1985). The simplest way to approximate a homogeneous distribution is to place particles at the vertices of a regular grid. Since the regularity of the grid obviously induces unwanted long-range correlations between the particles, another computationally more demanding way of producing an approximately homogeneous mass distribution has been widely used. This technique involves scattering particles at random over the computational volume, and then

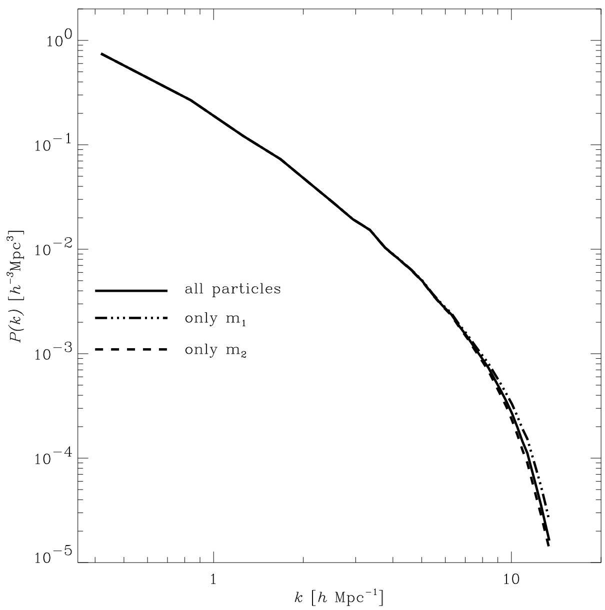

Figure 1. Power spectra of the initial particle distributions.

moving them for some time under the influence of a fictitious repulsive gravity. The motion of particles away from one another flattens out Poissonian density inhomogeneities of the initial distribution to produce a rather homogeneous glass (White, 1996).

We have initialized simulations containing two particle species using both of the above approaches to the production of an approximately homogeneous distribution. Unfortunately, glasses containing two particle species contain complex short-range correlations because the size of the void around each particle that is generated by repulsive gravity depends on the mass of the particle. Consequently, multi-mass glasses have complex and mass-dependent short-range correlations that obscure the effects of two-body relaxation. Since initial conditions based on a regular lattice yield much cleaner results, we confine the discussion to these simulations. The results obtained with glass-initialized simulations are consistent with the results presented here.

Our lattice based simulations were initialized as follows. 64^3 particles of one species were placed on the nodes of a regular grid with 64^3 cells covering a physical scale of $15h^{-1}$ Mpc on a side. Then 64^3 particles were placed on the nodes of the grid that is offset from the first grid by half a lattice spacing parallel to each axis. Finally Zel'dovich displacements were applied to each mass. In the final configuration, the particles of a given species define a density field that differs only in its overall normalization from the density field defined by particles of the other species. Hence the composite density field defined by both species together is also the same, up to an overall normalization. The masses of individual particles were set such that the composite density was that of the standard CDM model and the masses of the two species were in the ratio $1:\sqrt{2}$ or $1:4$. It follows that the more massive species accounted for 59 per cent and 80 per cent of the total mass in the two cases. Fig. 1 shows the power spectra of the initial particle distribution. The power spectra of the light (m_1) and the heavy particles are indis-

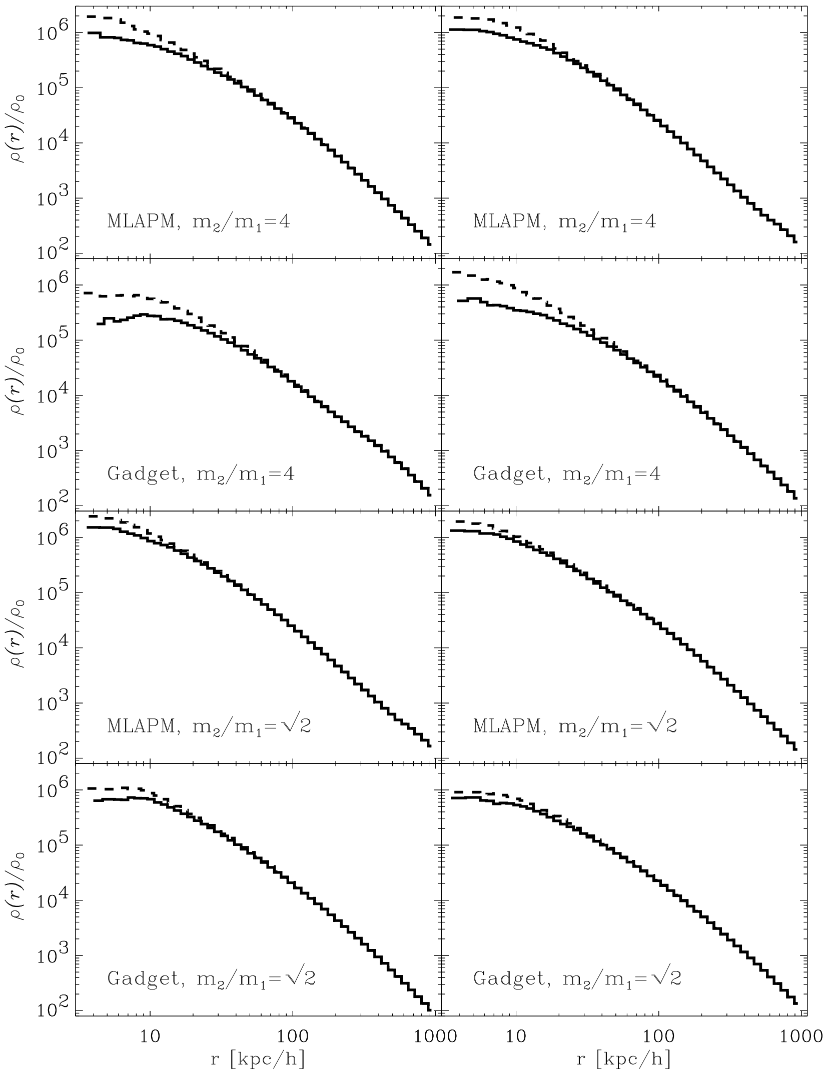

Figure 2. Density profiles of the two most massive halos in four simulations. Each column shows results for one of the halos. Full curves show the number density of the less massive particles, and dashed curves show the number density of the more massive particles.

tinguishable except at the highest wavenumbers, where the offset between the lattices from which the particles started gives rise to very slight differences in the way in which artifacts arising from the lattices impact on the power spectrum.

3 RESULTS

We evolved the particle distributions that are described by Fig. 1 in a standard CDM cosmology from redshift $z = 35$ to the current epoch using both the tree code GADGET (Springel et al. 2001) and our own multigrid code, MLAPM (Knebe et al. 2001). The essential difference between these codes is that whereas GADGET uses a spatially invariant gravitational softening length, MLAPM softens gravity adaptively so as to provide a compromise between enhanced force resolution and reduced particle noise in the force-field. The GADGET's Plummer softening length was set to $\epsilon = \min[5, 1/a(t)]h^{-1}$ kpc, where $a < 1$ is the scale factor, so at late times the code used accurately Newtonian forces for distances greater than $5h^{-1}$ kpc. MLAPM operates by refining the grid on which it solves Poisson's equation whenever the density exceeds a critical value $\rho_{\rm ref}$, typically set to 8 particles per cell. For these experiments the density compared to $\rho_{\rm ref}$ was not the density of gravitating mass, but the number density of particles, irrespective of their mass. The finest grid employed at the end of the simulation had a mesh spacing equivalent to 8192^3 cells in the computational volume, each cell being $1.8h^{-1}$ kpc on a side. On such a mesh the force law becomes Newtonian for distances $\gtrsim 5h^{-1}$ kpc (Knebe et al., 2001). Hence the spatial resolution provided by the two codes agreed well in high-density regions.

The GADGET runs required 244 398 and 249 677 timesteps, while the MLAPM runs used the equivalent of 1000 timesteps on the finest domain grid (128 cells on a side), during which 64 000 timesteps would be taken on an 8192^3 grid.

Fig. 2 shows the number-density profiles of the less and more massive particles of the two most massive halos to form

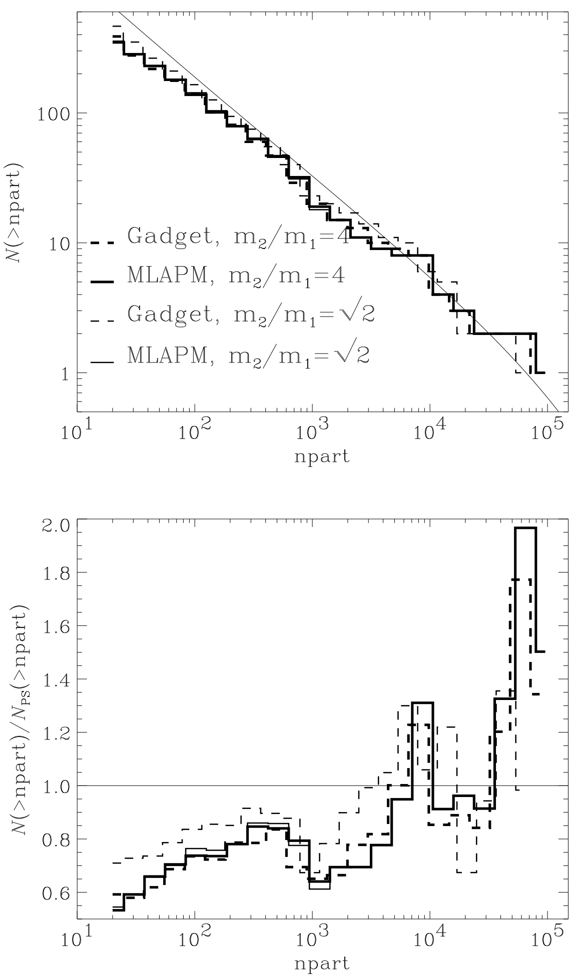

Figure 3. Mass functions from the simulations. The friends-of-friends algorithm has been used. The ordinate shows the number of particles in each object, not the total mass. The lower panel shows the ratio of the actual number of clusters to predicted by Press-Schechter theory, and the thin full curve in the upper panel shows this prediction.

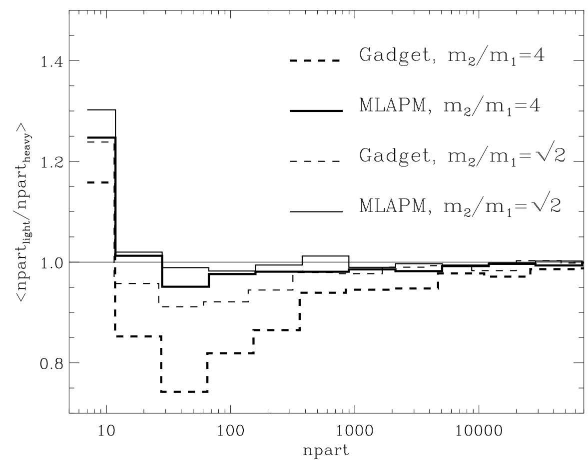

Figure 4. Ratio of the number of light to heavy particles in halos as a function of the total number of particles in the halo.

in the simulations. As we will see below in Fig. 4, both these halos contain about an equal number of light and heavy particles in every run. For each halo four pairs of profiles are shown, corresponding to the two mass ratios and two codes. Since the full curves, which show the profile of the less massive particles, always lie below the dashed curves, mass segregation is clearly detected. The effect is slightly more pronounced for the halo shown on the left than for that shown on the right. The main difference between the results obtained with the two codes is that MLAPM produces a central density that is about a factor 2 larger than that produced by GADGET, and mass segregation is more pronounced in the profiles produced by GADGET.

The effects of two-body relaxation on the less massive halos in the simulations is examined by Fig. 3, which shows for the four simulations the numbers of halos formed that contain in excess of a given number of particles (irrespective of particle mass). The thin full curve in the upper panel shows the prediction of Press-Schechter theory. In the lower panel we show for each species the ratio of the actual number of cluster to that predicted by Press-Schechter theory. The full histograms, which show the results obtained for the two mass ratios by MLAPM, are almost indistinguishable. The results obtained with GADGET agree moderately well with the MLAPM results only for $m_2/m_1 = 4$. The GADGET results for $m_2/m_1 = \sqrt{2}$ show significantly more low-mass clusters.

Fig. 4 clarifies what is going on by plotting as a function of the total particle number of a given halo the ratio of the number of light to heavy particles. Since in the simulation as a whole there are equal numbers of particles of each species, this ratio would ideally be unity, independent of total particle number. The results from MLAPM deviate significantly from this ideal only for $n_{\mathrm{part}} < 10$, where for both mass ratios clusters have more light than heavy particles. The results from GADGET, by contrast, reveal a significant tendency for clusters containing > 10 particles to have more heavy than light particles. The tendency is strongest for $m_2/m_1 = 4$, as would be expected if it were the result of two-particle relaxation causing heavy particles to sink towards the centres of clusters, while light particles evaporate from them. It is not clear why in all simulations clusters with only a handful of particles tend to have an excess of light particles, but this phenomenon may be connected with the fact that in Fig. 1 the power spectrum of the light particles is higher than that of the heavy particles at large k. We believe this phenomenon to be of no physical significance and to arise by chance from the phases of the waves used to displace the particles.

4 DISCUSSION

Simulations of cosmological clustering with two mass species have been used to explore the importance of two-body relaxation in general cosmological simulations. In the two largest halos that form in our simulations, mass segregation is a small but detectable effect in the sense that the difference in the central densities of particles with masses in the ratio 4 : 1 is comparable to the difference in the central densities

obtained for any one species with the two codes (about a factor 2). Mass segregation is stronger in simulations run with GADGET, which has (spatially) fixed softening, than with MLAPM, which softens adaptively.

In the MLAPM simulations there is no evidence that two-particle relaxation enhances the fraction of heavy particles in clusters. The GADGET simulations show a clear tendency for clusters with more than a handful of particles to have more massive than light particles. This tendency increases in strength with the mass ratio m_2/m_1, just as is expected if it is driven by two-particle relaxation.

Given the clear indication that two-particle relaxation is more pronounced in the GADGET simulations, it is interesting that it is in the MLAPM simulations that the central densities of the two most massive clusters are largest (by about a factor 2).

The major difference between the GADGET and MLAPM simulations was one of computational cost: using a small softening length everywhere increases the time required to run a simulation, both because it increases the cost of evaluating forces, and, more importantly, because it forces one to smaller timesteps. Quantitatively, the present GADGET simulations took longer than the corresponding MLAPM simulations by a factor about three. Since the quality of the scientific output from an N-body simulation is always limited by the number of particles employed, and most simulators are more limited by the availability of computer time than computer memory, the different speeds of our simulation amounts to a strong case for the use of adaptive softening of the type that MLAPM, and certain tree codes (Dehnen 2000) provide.

The equipartition time, on which mass segregation develops, is faster than the two-body relaxation time by roughly the ratio of the particle masses. By contrast, the core-collapse timescale, on which the central density of a halo is increased by two-body relaxation, is typically tens to hundreds of two-body relaxation times (e.g., Binney & Tremaine 1987). Hence, our finding that mass segregation is a marginal effect for mass ratio $4:1$, strongly suggests that the cuspiness of dark-matter halos in simulations is not an artifact produced by two-body relaxation. This conclusion is reinforced by the observation that central densities are highest in simulations in which mass segregation is least evident.

If the cuspiness of halos is not caused by high-k power in the input spectrum, as experiments with WDM suggests (Knebe et al. 2002), and not produced by two-body relaxation as we have argued, what *is* its origin? It must result from violent relaxation: in the absence of high-k power, the quasi-linear regime will continue until massive objects are ready to virialize. These will collapse by large factors because they will collapse from smooth and symmetrical initial conditions. Numerical experiments long ago established that the peak density in the final virialized entity is comparable to the peak density achieved during collapse (Doroshkevich & Klypin 1981; van Albada 1982). Hence, there is an argument for less high-k power giving rise to higher central densities.

In the presence of high-k power, violent relaxation will be unimportant for the formation of massive objects: these will form hierarchically through a series of mergers, and their cores are likely to be dominated by material that virialized early on as part of a low-mass object. The insensitivity of density profiles to the initial power spectrum must be the result of a conspiracy, which ensures that as high-k power is smoothed away, the loss of high-density seeds from early virialization is counteracted by an increase in the effectiveness of violent relaxation at late times. The bottom line is that cuspy density profiles must be considered inevitable so long as structure formation is dominated by a collisionless fluid.

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